A Note on Curve Plotting

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The use of the time constant of a particular circuit as a means of expressing the shape of a curve is becoming more common. Here's how to use this technique.

THE LITERATURE OF ELECTRONICS and audio engineering often employs the concept of time constant (of a particular circuit) for describing the response curve of an amplifier. Some readers may not be familiar with this concept, or else, may not wish to take out the time in making a whole series of calculations (depending on the number of circuits involved in the description of the curve). It is the purpose of the following to clarify this concept of time constant and to provide a set of curves which quickly provide the required information.

As an example consider the following text, taken from the EIA Standard RS-288 "Magnetic Playback Characteristic at 7½ ips:"

> 2.2.1 IN GENERAL, a curve that falls with increase of frequency at the rate of 6 db per octave, but modified—
> 2.2.2. AT LOW FREQUENCIES, be a curve that falls with decrease in frequency in conformity with the admittance of a series combination of a capacitance and a resistance having a time constant of 3180 microseconds, and further modified—
> 2.2.3. AT HIGH FREQUENCIES, by a curve that rises with increase in frequency in conformity with the admittance of a parallel combination of a capacitance and a resistance having a time constant of 50 microseconds—
> 2.2.4 THE COMPOSITE CURVE is shown in Figure 6 of the N.A.B. Recording and Reproducing Standards Supplement #2 dated June 1953.

There is obviously no problem connected with the first part of the above text which calls for a curve that falls with increase of frequency at the rate of 6-db-per-octave. The second paragraph, however, may present a problem to some, particularly when no guiding curve is supplied with the text, as

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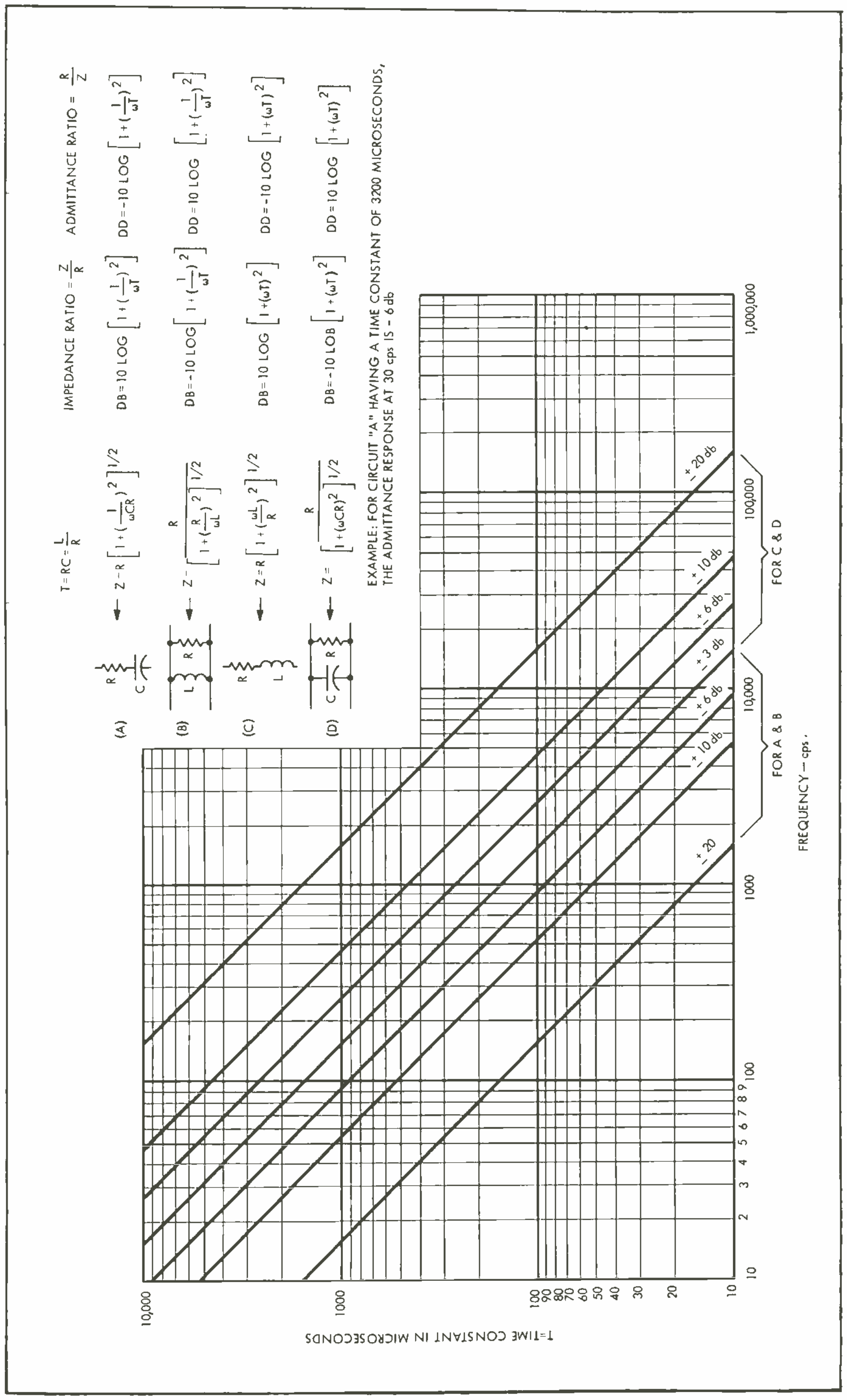

Fig. 1. Curve showing the time constant of various circuits at a particular frequency.

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was the case with the particular standard. What is called for is a curve conforming with the admittance of a series combination of a capacitance and a resistance having a certain time constant. Since the desired curve is one expressed in decibels, one has to consider not pure admittances, but ratios of admittances, as follows:

Circuit "A" of *Fig.* 1 shows a series R-C circuit whose absolute value of impedance is given by:

$$Z = R\left[1 + \left(\frac{1}{\omega CR}\right)^2\right]^{\frac{1}{2}} \quad Eq.\ (1)$$

The time constant, T, of the discharge of a capacitor through a resistor is defined as the time required for the voltage or current to decay to $1/e = 0.3679$ of its value. For the charge of a capacitor, the same definition applies, the voltage "decaying" toward its steady-state value. The time constant of discharge or charge of the current in an inductor through a resistor follows a similar definition. Without going through the oft-published computations for these circuits, we may write

$$T = CR = \frac{L}{R} \quad Eq.\ (2)$$

Substituting *Eq.* (2) in (1) we obtain

$$Z = R\left[1 + \left(\frac{1}{\omega T}\right)^2\right]^{\frac{1}{2}}$$

To plot the impedance variation with frequency in decibel fashion, we must know the impedance at a frequency when the reactive element has a negligible effect, that is, when the resistance is the controlling element. This we may do by writing

$$\text{db} = 20 \text{ lag } \frac{Z}{R} = 10 \log \left[1 + \left(\frac{1}{\omega T}\right)^2\right]$$

The equation for the admittance response, expressed in db, is the same as the equation above, except for a minus sign, because the admittance is the reciprocal of the impedance.

What remains to be done is to plot the curve for the time constant $T = 0.00318$ seconds as given by

$$\text{db} = -10 \log \left[1 + \left(\frac{1}{2\pi f \times 0.00318}\right)^2\right]$$

We may similarly plot the admittance response for a parallel combination of a capacitance and a resistance having a time constant of 50 microseconds to achieve the desired high-frequency response. The values can be obtained directly from the curves of *Fig.* 1 and introduced in the field of *Fig.* 2. The solid line of *Fig.* 2 represents the modification of the 6-db-per-octave curve in accordance with the prescribed test for the magnetic playback characteristics of ¼ in. tape travelling at 7½ inches per second.

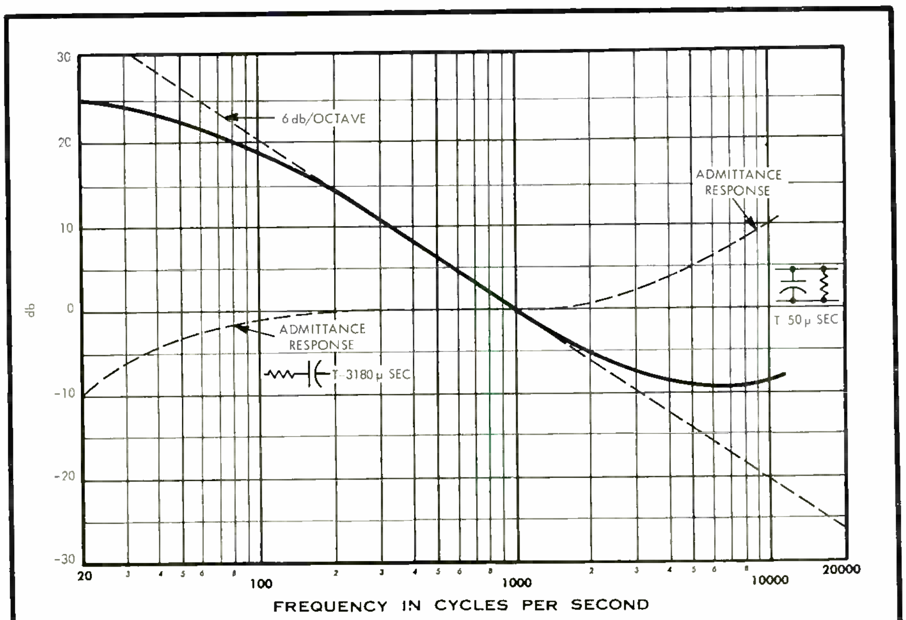

Fig. 2. Composite curve.